Electromagnetic theory

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3$^{\mathrm{rd}}$ SMRT Training School, AWI, 06-08 July 2023

Outline

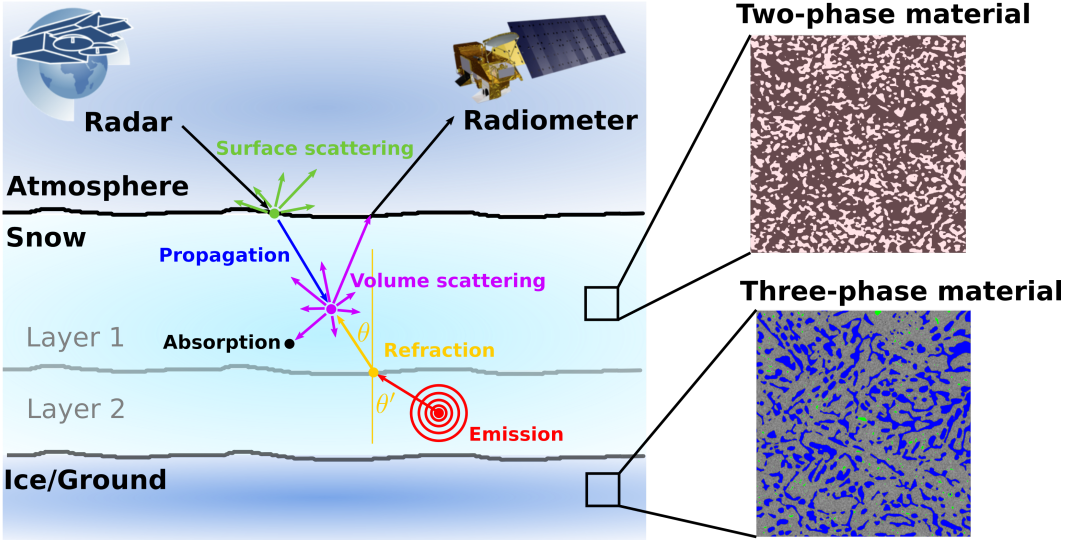
Introduction

Wave propagation, effective permittivity

Volume scattering, phase function, absorption

Interfaces, surface scattering

The problem at a glance: RS of snow and ice



The link to SMRT

SMRT's task:

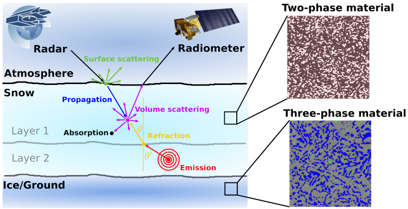
- Solving the radiative transfer equation (RTE):

$$\mu \frac{\partial \mathbf{I}(\mu,\phi,z)}{\partial z} = -\boldsymbol{\kappa}_{\mathrm{e}}(\mu,\phi,z)\,\mathbf{I}(\mu,\phi,z) + \frac{1}{4\pi}\iint_{4\pi} \mathsf{P}(\mu,\phi;\mu',\phi',z)\mathbf{I}(\mu',\phi',z)\,d\Omega' + \boldsymbol{\kappa}_{\mathrm{a}}(\mu,\phi,z)\,\alpha T(z)1$$

for the Stokes vector $\mathbf{I}$.

Our task:

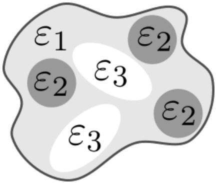
- Providing electromagnetic material properties for snow, ice in the RTE above (P, $\boldsymbol{\kappa}_{\mathrm{e}}$,$\boldsymbol{\kappa}_{\mathrm{a}}$) and making optimal choices in view of the picture on the right.



The systematic way of doing this

Take a random dielectric material:

$$\varepsilon(\boldsymbol{r}) = \begin{cases} \varepsilon_1 \text{ if } \boldsymbol{r} \text{ is in air} \\ \varepsilon_2 \text{ if } \boldsymbol{r} \text{ is in ice} \\ \varepsilon_3 \text{ if } \boldsymbol{r} \text{ is in brine} \end{cases} .$$



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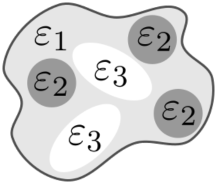

Solve Maxwell's equation

for the micro-scale electric field $\boldsymbol{E}$

(1) $$\nabla \times \nabla \boldsymbol{E}(\boldsymbol{r}) - \frac{k_0^2}{\varepsilon_0}\varepsilon(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r}) = 0$$

(vacuum wave number $k_0 = 2\pi\nu/c_0$, frequency ν, speed of light c_0)

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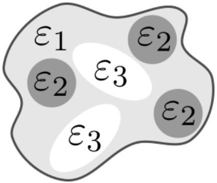

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(vacuum wave number $k_0 = 2\pi\nu/c_0$, frequency ν, speed of light c_0)

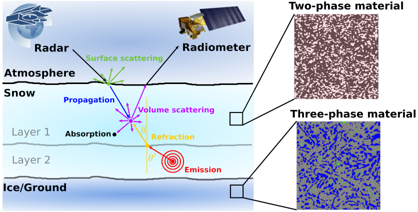
Derive effective EM properties

from the solution by volume averaging $\rightarrow$ **ALL** properties inherit from microstructure

The practical way of doing this

The common way of building/using models:

- Collecting available formulations for EM properties
- Mixing approximations that involve different assumptions
- Hoping for the best



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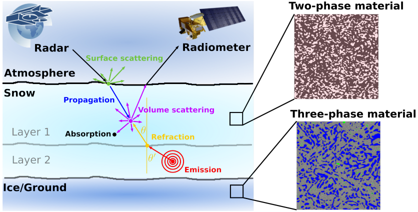

Goal of the lecture:

- Understanding involved EM processes and properties
- Understanding involved assumptions

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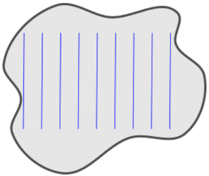
Propagation of plane waves in a homogeneous material

Plane waves:

If the permittivity ε does not depend on position (homogeneus), Eq. (1) admits plane wave solution:



$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \exp(ik\widehat{\boldsymbol{k}} \cdot \boldsymbol{r} - i\omega t)$$

with a complex **propagation constant** k

$$k = k' + ik''$$

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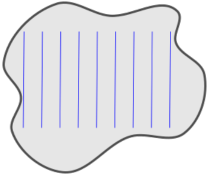

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$$k \quad = \quad nk_0$$

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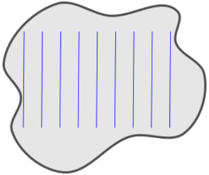

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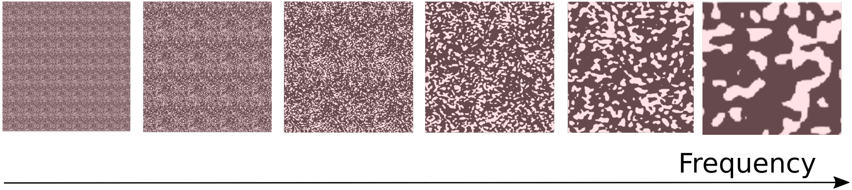
which is in turn related to the complex **dielectric constant** ε (or **permittivity**)

$$\varepsilon = n^2$$

All quantities k, n, ε are equivalent, complex-valued, EM material properties.

Propagation of plane waves in homogeneous snow or ice

When is snow or ice a homogeneous medium?



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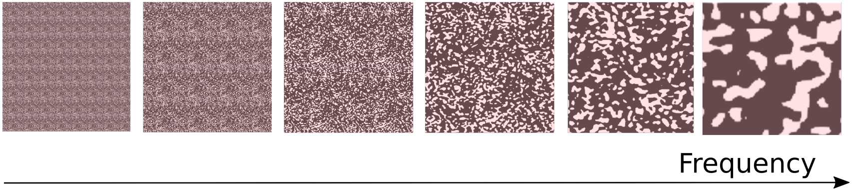


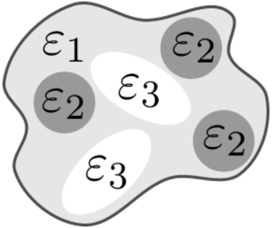
For "very low" frequency:

- Snow or saline ice can be regarded as a homogeneous medium described by an **effective permittivity** (rigorous concept)
- Effective permittivity contains microstructure only via volume fractions ("grain size" does not enter).

Effective permittivity of mixtures (air, ice, water, brine)

How effective permittivities are mostly derived:

- Place randomly oriented spheroids with permittivity $\varepsilon_2, \varepsilon_3$ in a background medium ε_1.



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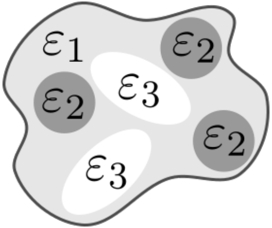

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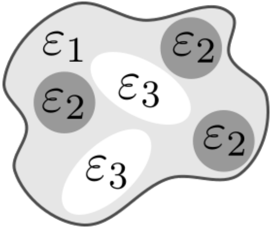

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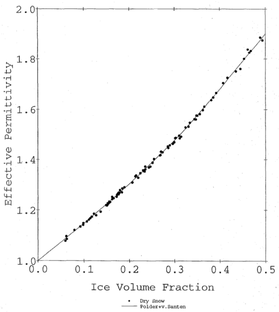
Permittivity formulations in SMRT:

(cf. `smrt.permittivity`)

- Polder–van Santen (default)
- Bruggemann
- Maxwell–Garnett
- $+$ many others



[MÄTZLER 1996]

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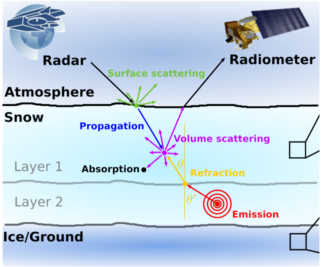
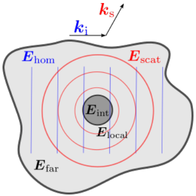


Recap: Single sphere scattering

Heterogeneities:

Perfect homogeneity is an idealization valid only for $k \to 0$. By increasing the frequency, the plane wave will start to “see” heterogeneities

When a plane wave hits a sphere, the solution of Eq. (1)



$$\boldsymbol{E} = \boldsymbol{E}_{hom} + \boldsymbol{E}_0 f(\boldsymbol{k}_s, \boldsymbol{k}_i) \frac{\exp ikr}{r} \qquad (2)$$

superposes background field (hom) and scattered field (scat)

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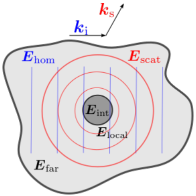

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Size-frequency scaling:

- $ka \ll 1$: Rayleigh, $ka \approx 1$: Mie, $ka \gg 1$: Geometrical optics

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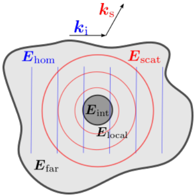

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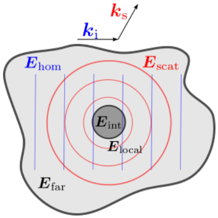
Energy conservation during scattering:

- Scattering coefficient, absorption coefficient, phase function and dielectric constant are all linked

Relation between scattering quantities

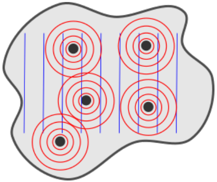
Phase function	$p = \frac{4\pi}{\kappa_s} \lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i) \rvert^2$	Angular distribution of scattered intensity
Scattering coefficient:	$\kappa_s = \frac{1}{4\pi} \int_{4\pi} d\Omega \lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i) \rvert^2$	Total scattered intensity
Extinction coefficient:	$\kappa_e = 2\mathcal{I}m(\sqrt{\varepsilon_{eff}})$	Intensity attenuation (including scattering and absorption)
Absorption coefficient:	$\kappa_a = \kappa_e - \kappa_s,$	Intensity attenuation due to Ohmic currents

Table 1: Interrelation of scattering quantities



Scattering approximations for random two-phase media:

The Maxwell Eq (1) can be written as a perturbation scheme



$$(3) \qquad \nabla \times \nabla \boldsymbol{E}(\boldsymbol{r}) - \frac{k_0}{\varepsilon_0}\varepsilon_{hom}\boldsymbol{E}(\boldsymbol{r}) = \frac{k_0}{\varepsilon_0}[\varepsilon(\boldsymbol{r}) - \varepsilon_{hom}]\boldsymbol{E}(\boldsymbol{r})$$

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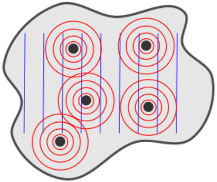

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The relevance of ε_{hom}

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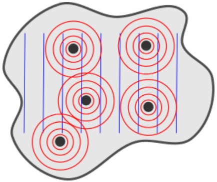

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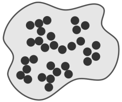
Scattering approximations in SMRT:

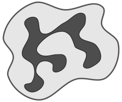
(cf. `smrt.emmodel`)

- QCA: Quasicrystalline approximation
- QCA-CP: Quasicrystalline approximation (coherent potential)
- SFT: Strong fluctuation theory
- IBA: Improved Born approximation
- SCE: Strong contrast expansion

QCA, QCA-CP, IBA, SCE:



IBA, SFT, SCE:



The improved Born approximation (IBA): Believing by analogy

Rayleigh phase function in the IBA:

$$f_{scat}(\chi) \sim M(|\mathrm{k}_d|)\; k^4\; \sin^2\chi\; F_{\mathrm{IBA}}(\varepsilon_1, \varepsilon_2)$$

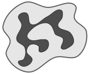
Rayleigh phase function of a sphere:

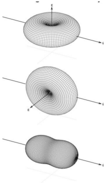
$$f_{scat}(\chi) \sim \qquad a^6\; k^4 \sin^2\chi\; F_{\mathrm{sph}}(\varepsilon_1, \varepsilon_2)$$

- Size term
- Angle term
- Dielectric term

- We just need a generalized understanding of size ($\rightarrow$ tomorrow)

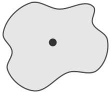
Arbitrary bicontinuous:





[SLIDEPLAYER/ROGERS]

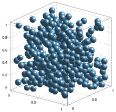
Small sphere:

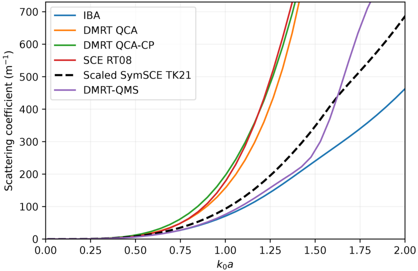


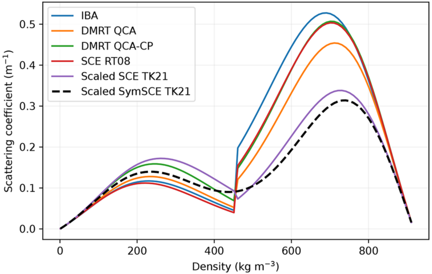
Comparison of scattering formulations for two-phase media (snow)

For a sticky hard sphere microstructure

(varying density, varying radius *a*)







[Picard et al, TC, 2022]

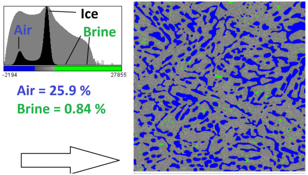
Scattering formulations:

- Differences grow as a function of k_0a
- Continuity as a function of density requires “symmetrization”

Scattering in three-phase media (sea ice)

Scattering formulations for three phase media

- In principle: Restart from Eq. (3) and redo Tab. 1
- Such a consistent approach is presently lacking
- SMRT: Pragmatic combination of effective permittivity, inclusions shape and IBA two-phase scattering, discontinuous!



(Maus et al, TC, 2021)

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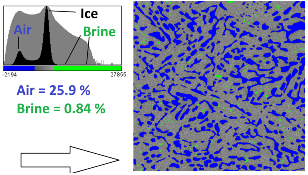

(Maus et al, TC, 2021)

First year ice:

- Spheroidal brine inclusions in a pure ice background

Multi year ice:

- Spheroidal air inclusions in a saline ice background

Getting back from scattering to the RTE

The RTE in SMRT considers all Stokes components

$$\mathbf{I} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} |\boldsymbol{E}_\mathrm{H}|^2 + |\boldsymbol{E}_\mathrm{V}|^2 \\ |\boldsymbol{E}_\mathrm{H}|^2 - |\boldsymbol{E}_\mathrm{V}|^2 \\ 2\mathcal{R}e(\boldsymbol{E}_\mathrm{H}\boldsymbol{E}_\mathrm{V}*) \\ 2\mathcal{R}e(\boldsymbol{E}_\mathrm{V}\boldsymbol{E}_\mathrm{H}*) \end{bmatrix}$$

The 4×4 phase matrix:

$$\mathrm{P}(\mu, \phi, \mu', \phi') = \begin{bmatrix} P_{11} & P_{12} & P_{13} & 0 \\ P_{21} & P_{22} & P_{23} & 0 \\ P_{31} & P_{32} & P_{33} & 0 \\ 0 & 0 & 0 & P_{44} \end{bmatrix}$$

- can be computed from $f_{scat}(\chi)$ (details in Picard et al 2018)

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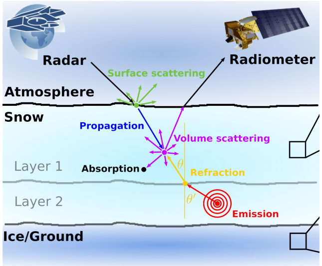
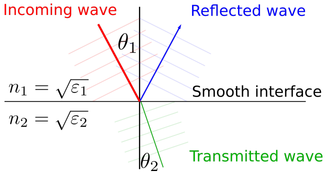


Plane waves at smooth interfaces



Refraction:

- Snells law:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

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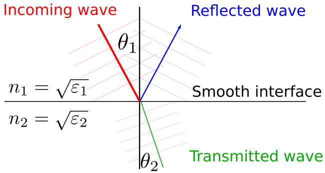

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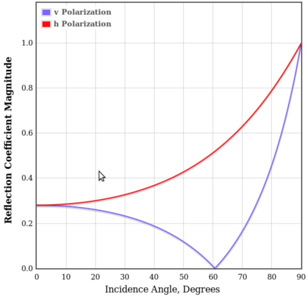
Transmission and reflection:

- Fresnel formulas
- Angles, intensities determined by the effective permittivities ε_1 ε_2 and polarization
- Whether a surface is smooth depends on k

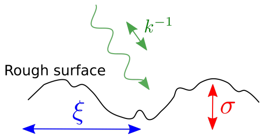


[Ulaby 2013]

Plane waves at random rough interfaces

Geometrical characterization:

- Involves at least **two length scales**
- Vertical height standard deviation σ
- Horizontal correlation lenght ξ



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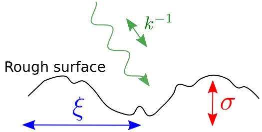
- Vertical height standard deviation σ
- Horizontal correlation lenght ξ



EM surface scattering theory:

- Involves at least **two lenght scale ratios**, either $(k\sigma,\ k\xi)$ or $(k\sigma,\ \sigma/\xi)$
- Approximations depend on their magnitude

Plane waves at random rough interfaces

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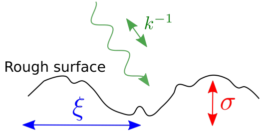

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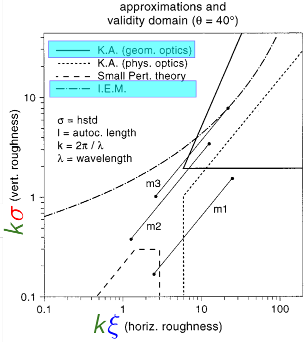

[Adapted from Macelloni 2000]

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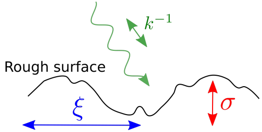

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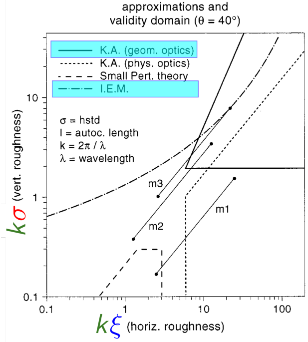


[Adapted from Macelloni 2000]

Surface scattering models in SMRT

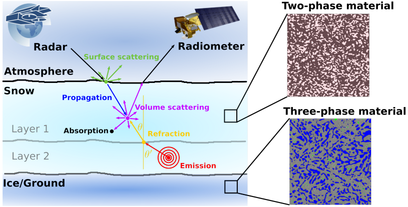
(`smrt.interface`)

- Geometrical optics $k\sigma \gg 1$, $k\xi \gg 1$, σ/ξ fixed
- Integral equation method (IEM)

Summary: Electromagnetic theory

EM ingredients in SMRT

- RTE needs P, $\boldsymbol{\kappa}_{\mathrm{s}}$, $\boldsymbol{\kappa}_{\mathrm{a}}$, $\boldsymbol{\varepsilon}_{\mathrm{eff}}$
- Many available formulations implemented
- Many carefully chosen default options



Thank you for your attention.